

Strategic Fertility and the Demographic Transition

Diogo Baerlocher (University of South Florida)

Stephen Parente (University of Illinois at Urbana-Champaign)

Henrique Veras (Universidade Federal de Pernambuco)

July 2026*

Abstract

We propose a strategic motive for fertility. When property rights are imperfectly enforced, larger populations increase a society's ability to defend valuable land from external appropriation, creating a strategic return to population alongside its productive role. As industrialization reduces the economic importance of land, this strategic return declines, providing a mechanism through which structural transformation may have contributed to the demographic transition. We evaluate this hypothesis using a panel of French departments spanning 1806–1911. Our empirical strategy exploits the 1871 annexation of Alsace-Lorraine, which unexpectedly shifted the Franco-German frontier westward and increased the strategic importance of several previously interior departments. Fertility rose by approximately 7 percent in the newly exposed departments, with no evidence of differential pre-treatment trends and robust results across alternative comparison groups. The increase is concentrated in departments where land remained relatively productive, consistent with the model's comparative statics, while broader cross-sectional evidence suggests that industrialization weakened this strategic fertility incentive over time. Together, the results identify a previously overlooked strategic determinant of fertility and suggest that its gradual disappearance contributed to the historical fertility transition.

Keywords: Fertility; Demographic Transition; Malthusian Stagnation; Territorial Conflict; Expropriation Risk; Unified Growth Theory. **JEL Codes:** J13; N33; O41; D74.

*Baerlocher: University of South Florida, Department of Economics, 4202 E. Fowler Ave. CMC342, Tampa, FL, 33620 (email: baerlocher@usf.edu). Parente: University of Illinois at Urbana-Champaign, Department of Economics, 214 David Kinley Hall, 1407 W. Gregory Dr., Urbana, IL, 61801 (email: parente@illinois.edu). Veras: Universidade Federal de Pernambuco, Department of Economics (email: henrique.fonseca@ufpe.br).

1 Introduction

Why did pre-industrial societies sustain high fertility for centuries, and why did fertility begin to decline during industrialization? Existing explanations of the demographic transition largely locate the answer within the household. Fertility falls because the return to human capital rises and parents substitute child quality for quantity [Becker et al., 1990, Galor and Weil, 2000], because the opportunity cost of children increases, or because declining mortality alters the optimal size of the family. A common feature of these explanations is that changes in fertility are driven by changes in the private returns to childbearing.

This paper proposes a complementary mechanism. We argue that, in agrarian societies with imperfectly enforced property rights, fertility responded not only to private household incentives but also to a societal incentive to maintain population. Population was a strategic asset: larger populations increased a society's ability to defend productive land against external appropriation [North and Thomas, 1973, Alesina and Spolaore, 2003]. We refer to this as a *strategic motive for fertility*. Although the idea that population contributes to territorial security is not new—Parente and Prescott [2005], for example, develop this intuition in a theory of international income differences—its implications for fertility and long-run demographic change have not been formally developed or empirically evaluated.

The mechanism rests on two simple observations. First, the strategic value of population depends on the value of the land it helps defend. Second, this strategic incentive exists only when land is vulnerable to external appropriation. Together, these conditions imply that the strategic return to fertility is strongest where land is both economically important and contestable. Industrialization weakens this incentive by reducing the importance of land in production, suggesting a novel channel through which structural transformation may have contributed to the historical decline in fertility.

To organize these ideas, we develop a simple extension of Hansen and Prescott [2002] in which two societies bargain over a fixed stock of land and a failed bargain leads to conflict, with the probability of victory increasing in relative population. The model delivers the empirical predictions that guide the analysis: an increase in territorial contestability raises fertility, and this response is strongest where land remains central to production.

Our empirical contribution is to identify this strategic fertility motive using one of the sharpest geopolitical shocks in nineteenth-century Europe. The annexation of Alsace-Lorraine in 1871 unexpectedly shifted the Franco-German frontier approximately one hundred kilometres westward. Departments that had previously been shielded by Alsace and Lorraine became frontier departments overnight, while the remainder of France experienced no comparable change in territorial exposure. This boundary reassignment generated plausibly exogenous variation in the strategic value of land, allowing us to estimate the causal effect of increased territorial contestability on fertility.

The evidence strongly supports the existence of a strategic fertility motive. We estimate that fertility increased by approximately seven percent in departments newly exposed to the frontier relative to the rest of France. The estimates are remarkably stable across alternative specifications, comparison groups, and estimation methods, including matched samples and synthetic controls. Event-study estimates reveal no evidence of differential pre-treatment trends and show that the increase emerged immediately after the frontier shifted and persisted through the end of our sample.

The historical experiment also provides direct evidence on the mechanism. The fertility response is concentrated in departments where land remained relatively productive, precisely where the model predicts that territorial contestability should matter most. By contrast, de-

partments where land had become less important exhibit little or no response to the increase in expropriation risk. We also document broader cross-sectional patterns consistent with the model’s comparative statics. Industrialization is associated with lower fertility throughout the sample, and the estimated interactions between industrialization, land dependence, and historical contestability uniformly carry the signs implied by the theory, although they are estimated imprecisely. We interpret these results as suggestive evidence that the strategic fertility channel identified by the natural experiment may also have contributed to the broader historical fertility decline.

A natural alternative explanation is that the observed increase reflects changes in population composition rather than fertility behavior. The annexation displaced a substantial number of refugees from Alsace-Lorraine, many of whom settled in the departments adjacent to the new frontier. The data provide little support for this interpretation. Refugee intensity has no independent association with fertility, controlling for it increases rather than attenuates the estimated frontier effect, and the dynamic response of refugee settlement differs markedly from that of fertility.

Taken together, the empirical evidence serves two complementary purposes. The 1871 frontier experiment establishes that exogenous increases in the strategic value of land raise fertility, providing direct evidence for the strategic fertility mechanism. The broader cross-sectional evidence then asks whether the process of industrialization exhibits the comparative statics implied by that mechanism. Although these latter results are necessarily more suggestive than the historical experiment itself, they consistently point in the direction predicted by the theory and indicate that the gradual decline in the strategic value of land may have been one factor contributing to Europe’s fertility transition.

Our results contribute to three strands of the literature. First, we contribute to the unified growth literature [Galor and Weil, 2000, Hansen and Prescott, 2002] by introducing a strategic mechanism linking structural transformation to long-run fertility decline. Rather than emphasizing changes in the productive return to children, our mechanism operates through changes in the strategic return to population. Second, we contribute to the literature on strategic fertility and population races [De la Croix and Dottori, 2008, Lagerlöf, 2010] by showing that the strategic value of population depends endogenously on the economic importance of land, implying that the incentive to sustain large populations diminishes naturally as economies industrialize. Finally, we contribute to the empirical literature on the historical determinants of fertility [Coale and Watkins, 1986, Daudin et al., 2019] by providing causal evidence that exogenous changes in territorial contestability influence fertility behavior.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3.1 describes the data. Section 3.2 presents the evidence from the 1871 frontier experiment. Section 3.5 examines whether broader patterns of industrialization are consistent with the model’s comparative statics. Section 4 concludes.

2 Model

This section develops the mechanism underlying the empirical analysis. Our objective is not to construct a fully fledged model of the demographic transition, but rather to isolate the strategic channel emphasized throughout the paper. We therefore keep the environment deliberately parsimonious, extending Hansen and Prescott [2002] by allowing the value of land to depend on the outcome of territorial competition between societies. The model delivers two empirical implications. First, an increase in territorial contestability raises fertility by increasing the strategic value of population. Second, because the value of defending territory depends on

how important land remains in production, this response weakens as industrialization shifts production away from land-intensive activities.

2.1 Environment

Time is discrete. Two societies, indexed as domestic and foreign, inhabit a world endowed with a fixed stock of land normalized to one. A tilde denotes foreign variables throughout. Let $\theta_t \in (0, 1)$ denote the domestic society's share of world land at date t , so that the foreign society holds the remaining share, $1 - \theta_t$. Apart from their interaction over land, the two societies are independent economies: they do not trade, and all strategic interaction occurs through the allocation of territory.

Production combines two constant-returns technologies. The first is a Malthusian technology that uses capital, labor, and land,

$$Y_t^M = A_{Mt} (z_{kt} K_t)^\phi (z_{ht} H_t)^\mu \theta_t^{1-\phi-\mu}, \quad (1)$$

where K_t denotes the capital stock, H_t aggregate labor input, and z_{kt} and z_{ht} are the fractions of capital and labor allocated to this sector. The second is a modern technology that combines only capital and labor,

$$Y_t^S = A_{St} [(1 - z_{kt}) K_t]^{1-\mu} [(1 - z_{ht}) H_t]^\mu. \quad (2)$$

The asymmetry between these two technologies is the key feature of the environment. Land is an essential input in the Malthusian sector but is absent from the modern sector. Competitive firms allocate capital and labor across sectors to maximize profits, implying that factor allocation depends on the relative productivity of the two technologies. As the ratio A_{St}/A_{Mt} increases, production shifts toward the modern sector and land becomes progressively less important. Throughout the paper, we refer to this compositional shift as *industrialization*. In the model, industrialization has no broader interpretation: it simply captures the declining productive importance of land.

2.2 Households

Individuals live for two periods. Preferences are defined over consumption when young and old and over the number of children,

$$\ln c_{1t} + \beta \ln c_{2t} + \eta \ln n_{t+1}, \quad (3)$$

where β is the discount factor and η captures the private value of children. Each young individual is endowed with one unit of time. Raising a child requires τ units of that time, so aggregate labor supply is

$$H_t = (1 - \tau n_{t+1}) N_t,$$

and the size of the next cohort is

$$N_{t+1} = n_{t+1} N_t.$$

Young individuals work, save in capital and land, and consume the resulting returns when old.

The key departure from standard demographic models is that fertility is chosen at the societal rather than the household level. This reflects the central externality emphasized in the paper. Individual households internalize the private costs and benefits of childbearing, but they do not internalize the contribution of population to territorial defense. A societal planner therefore chooses fertility to maximize the welfare of the representative household while accounting for the strategic value of population. Following De la Croix and Dottori [2008], we model fertility as a collective rather than an individual decision.

Substituting the household's budget constraints and optimal saving decision into lifetime utility yields the indirect utility function, up to constants,

$$(1 + \beta) \ln(1 - \tau n_{t+1}) + (1 + \beta) \ln w_t + \beta \mathbb{E}_t \ln r_{K,t+1} + \eta \ln n_{t+1}, \quad (4)$$

where w_t denotes the wage and $r_{K,t+1}$ the return to capital. The first term captures the opportunity cost of childrearing through forgone labor supply, while the last captures the direct utility from children. The remaining term reflects the expected return to saving, which depends on the future division of land. It is through this channel that territorial contestability enters households' welfare and gives rise to the strategic motive for fertility developed below.

2.3 Land, conflict, and contestability

Land is valuable but imperfectly protected. When the two societies disagree over its allocation, they may resort to conflict. We assume that the domestic society prevails with probability

$$\pi_t = \frac{N_t}{N_t + \tilde{N}_t}, \quad (5)$$

where N_t and $\tilde{N}_t$ denote the domestic and foreign populations. Population is therefore an input into territorial defense: larger societies are more likely to secure control over productive land. For simplicity, the winner obtains the entire disputed territory, so π_t also represents the expected territorial share under conflict.

Rather than fighting in equilibrium, societies bargain over the division of land. With equal bargaining power, each side must receive at least its expected payoff from conflict. The negotiated allocation therefore coincides with the expected conflict outcome,

$$\theta_{t+1} = \pi_{t+1}, \quad (6)$$

implying that a larger population translates into greater expected territorial control. If land no longer affects production, however, changes in territorial ownership do not alter economic returns, and the existing allocation is sustained without renegotiation.

Territory is not contested every period. Let $\rho \in [0, 1]$ denote the probability that the territorial allocation is renegotiated. With probability ρ , the division is determined by the bargaining outcome above; with probability $1 - \rho$, each society retains its existing territory. We treat ρ as an exogenous feature of the geopolitical environment. Empirically, it captures territorial contestability, and the 1871 annexation of Alsace–Lorraine provides precisely the type of exogenous increase in ρ studied in Section 3.2.

2.4 The strategic value of land

The benefit of controlling additional territory depends on how much land contributes to production. We summarize this dependence by the elasticity of the return to capital with respect to the

domestic land share,

$$\lambda_t \equiv \frac{\partial \ln r_{K,t+1}}{\partial \ln \theta_{t+1}}, \quad (7)$$

which we refer to as the *strategic value of land*. The parameter λ_t measures how much the return to productive assets changes when a society controls a larger share of territory. When λ_t is high, land is worth defending because changes in territorial control have substantial economic consequences. When λ_t is low, the same territorial changes matter little for production.

The production structure determines the magnitude of λ_t . If all production takes place in the Malthusian sector, the return to capital inherits the land share of production from equation (1), implying $\lambda_t = 1 - \phi - \mu$. At the opposite extreme, if production is entirely modern, the return to capital is determined by equation (2), which does not depend on land, so $\lambda_t = 0$. Since firms shift capital and labor toward the modern sector as its relative productivity A_{St}/A_{Mt} rises, industrialization gradually reduces λ_t from the Malthusian land share toward zero.

This parameter is the key object in the model. Population affects welfare because it increases the probability of retaining productive land, but that channel is valuable only to the extent that land itself contributes to production. As industrialization reduces the economic importance of land, it simultaneously reduces its strategic importance. Once λ_t approaches zero, changes in territorial control no longer affect the return to capital, the incentive to compete for land disappears, and so does the strategic motive for maintaining a large population.

2.5 Fertility and the trade-off

Substituting the bargained land allocation into equation (4) and taking expectations over territorial contestation, the society's objective can be written, up to constants, as

$$U(n_t, \tilde{n}_t) = \underbrace{\Phi \ln(1 - \tau n_t)}_{\text{time cost}} + \underbrace{(\beta\mu + \eta) \ln n_t}_{\text{private return}} + \underbrace{\beta\rho\lambda \ln \pi_{t+1}(n_t, \tilde{n}_t)}_{\text{strategic return}}, \quad (8)$$

where $\Phi \equiv \mu(1 + \beta\phi)$ summarizes the effect of child rearing on labor supply, wages, and capital accumulation.

The objective contains three components. Children reduce labor supply through their time cost, generate private utility, and increase the probability of retaining productive land. The strategic motive is governed by the product $\rho\lambda$: fertility responds to geopolitical competition only when land is both contested and economically valuable.

The objective is strictly concave, implying a unique interior optimum. In the symmetric equilibrium, both societies are of equal size, each prevails with probability one half, and fertility is

$$n^* = \frac{1}{\tau} \cdot \frac{Z}{\Phi + Z}, \quad Z \equiv \beta\mu + \eta + \frac{\beta\rho\lambda}{2}. \quad (9)$$

The strategic motive enters the fertility decision solely through the term $\beta\rho\lambda/2$. The empirical analysis therefore focuses on how changes in territorial contestability (ρ) and the productive importance of land (λ) shape fertility behavior.

2.6 Empirical implications

The model yields two empirical predictions that organize the remainder of the paper.

The first concerns territorial contestability. Fertility is increasing in the probability that land is contested, $\partial n^*/\partial \rho > 0$, because a greater risk of territorial loss raises the value of sustaining population. This response is stronger where land remains economically important, since the strategic return to population is proportional to $\rho\lambda$. We test this prediction using the 1871 annexation of Alsace-Lorraine as an exogenous increase in territorial contestability, and examine whether the response is concentrated in departments where land played a larger productive role.

The second concerns industrialization. Since industrialization reduces the productive importance of land, it lowers the strategic value of population and therefore reduces fertility. The model further predicts that this relationship should be strongest where land is historically more contested. We examine whether the observed relationship between industrialization and fertility is consistent with these comparative statics by interacting industrialization with land dependence and historical contestation.

Both predictions arise from the same mechanism. Population has a strategic value only when land is both economically valuable and vulnerable to expropriation. The demographic transition occurs as industrialization progressively weakens one of these conditions.

3 Empirical Analysis

This section evaluates the empirical implications of the model. We begin by describing the data and the construction of the key variables. We then exploit the 1871 annexation of Alsace-Lorraine as a sharp geopolitical shock that increased the strategic value of land for a subset of French departments, providing direct evidence on the relationship between territorial contestability and fertility. Finally, we examine whether the broader cross-sectional patterns implied by the model are present in the data, focusing on the interaction between territorial contestability, land dependence, and industrialization.

3.1 Data

Our empirical analysis combines a long panel of departmental fertility with measures of industrialization, land dependence, historical territorial contestability, and exposure to the new Franco-German frontier created in 1871. Unless otherwise noted, all variables are measured at the department level.

Together, these variables allow us to distinguish between evidence identifying the causal effect of an exogenous increase in territorial contestability and evidence assessing whether the broader comparative statics implied by the model are reflected in the cross-sectional variation across French departments.

Fertility. Our outcome variable is the departmental fertility index constructed by Bonneuil [1997] and distributed through the replication archive of Daudin et al. [2019]. The index is available for 83 French departments at quinquennial intervals between 1806 and 1911, yielding 1,768 non-missing observations across 22 periods. Throughout the paper we use the natural logarithm of the index, which both facilitates interpretation in proportional terms and accommodates its bounded support. Fertility declines substantially over the sample period, with the cross-sectional mean falling from 0.41 in 1806 to 0.24 in 1911, while cross-departmental dispersion narrows considerably.

1871 Frontier Exposure. We construct department boundaries as of 1790 from the French Historical GIS collection and measure each department's distance to the boundary of the territory annexed by Germany in 1871, comprising Bas-Rhin, Haut-Rhin and Moselle. Before 1871 this line was an internal administrative boundary; after 1871 it was an international frontier. We define as newly exposed the seven departments whose centroids lie within one hundred kilometres of it: Territoire de Belfort (9 kilometres), Meurthe-et-Moselle (26), Meuse (31), Vosges (48), Haute-Saône (52), Doubs (54) and Ardennes (59). The annexed departments themselves are excluded from all estimations.

Land Dependence. The model predicts that the strategic value of population depends on the productive importance of land rather than on the realized allocation of labor across sectors. We therefore proxy land dependence using agro-climatic suitability rather than agricultural employment, which is itself an equilibrium outcome. Specifically, we construct an index from the FAO GAEZ database by calculating area-weighted averages of low-input rainfed suitability for wheat, barley, oats, and rye over department polygons and averaging the standardized crop-specific indices.

Industrialization. Industrialization is measured by the share of the labor force employed in industrial occupations, obtained from the French population censuses. The series is available for six census years between 1851 and 1911.¹

Historical Territorial Contestability. We measure the historical contestability of a department's land by its exposure to pre-1700 interstate conflict, constructed as an inverse-distance kernel over the recorded locations of battles fought before 1700, excluding civil conflicts. Measuring contestation entirely before the sample period has two advantages: it cannot be contaminated by nineteenth-century industrialization, and it captures the historical strength of the defense motive rather than a contemporaneous strategic calculation, which is the construct the interpretation in Section 1 requires.

Refugee inflows. To address the composition alternative we measure in-migration from the annexed territories as a share of population, constructed as the difference between total recorded in-migrants and in-migrants excluding Alsace-Lorraine. Both series are reported in the Daudin et al. [2019] archive, and we use the 1901 cross-section as a departmental intensity.

Sample construction. The estimation sample differs across empirical exercises. The 1871 frontier analysis is based on 1,780 observations from 81 departments. It excludes the three annexed departments and requires an 1806 observation to construct the baseline fertility measure used in the convergence specification. The cross-sectional analysis in Section 3.5 is estimated on 472 observations, reflecting the availability of the industrial employment series only from 1851 onward. Sample sizes are reported in all tables.

A second issue concerns changes in administrative boundaries following the 1871 annexation. Neither Meurthe-et-Moselle nor the Territoire de Belfort existed as continuous administrative units throughout the sample period. Meurthe-et-Moselle was created from the French portion of the former department of Meurthe, while the Territoire de Belfort corresponds to the arrondissement of Haut-Rhin that remained under French control. To construct balanced departmental

¹We exclude the 1901 census because the reported industrial employment shares are inconsistent with the remaining waves and appear to reflect a data error rather than a definitional change.

series, we assign pre-1871 observations for Meurthe and Haut-Rhin to Meurthe-et-Moselle and the Territoire de Belfort, respectively. Because the latter involves the more substantial territorial discontinuity, we report robustness exercises excluding the Territoire de Belfort.

Taken together, the resulting dataset combines long-run variation in fertility with predetermined measures of land dependence and historical territorial contestability, as well as a discrete change in frontier exposure generated by the 1871 annexation. This combination allows us to separately examine the causal effect of a geopolitical increase in territorial contestability and the broader cross-sectional patterns implied by the model.

3.2 The 1871 Frontier Experiment

Our first empirical exercise exploits the annexation of Alsace–Lorraine following the Franco-Prussian War as a discrete geopolitical shock to the strategic value of land. The event is particularly well suited to our setting because it altered the expected cost of territorial loss without changing the underlying productivity of land. Agricultural suitability, climate, and the local production technology remained unchanged. What changed was the strategic environment: departments that had previously been buffered by Alsace and Lorraine became frontier regions facing the German Empire directly. The model predicts that such an increase in territorial contestability should raise fertility by increasing the strategic value of population.

Fig. 1 illustrates the resulting geography. Prior to 1871, the western boundary of Alsace–Lorraine was an internal administrative border with no implications for territorial security. Following the annexation, the same line became an international frontier separating France from the German Empire. Departments located immediately west of the annexed territory therefore experienced a discrete increase in frontier exposure, while the remainder of metropolitan France experienced no comparable change.

Identification therefore comes from comparing changes in fertility before and after 1871 between departments that became newly exposed to the frontier and departments whose geopolitical environment remained unchanged. Because department fixed effects absorb all time-invariant determinants of fertility—including land quality, historical territorial contestability, and persistent cultural differences—the coefficient of interest is identified exclusively from the change induced by the frontier reassignment.

The identifying assumption is that, absent the frontier reassignment, fertility in the newly exposed departments would have followed the same evolution as fertility elsewhere after conditioning on department and period fixed effects and differential convergence. We assess this assumption directly below using event-study estimates, matched comparisons, synthetic controls, and a range of robustness exercises.

We estimate the effect of this reassignment on fertility using the following specification:

$$\ln F_{rt} = \alpha_r + \delta_t + \beta (\text{Newly Frontier}_r \times \text{Post}_t) + X'_{rt}\gamma + \varepsilon_{rt}, \quad (10)$$

where F_{rt} is the fertility index in department r at period t , Newly Frontier_r is an indicator for the seven departments within one hundred kilometers of the frontier created in 1871, Post_t is an indicator for periods from 1871 onward, α_r and δ_t are department and period fixed effects, X_{rt} is a vector of controls, and ε_{rt} is the error term. The model predicts that an increase in territorial contestability raises the strategic value of population and therefore increases fertility, implying $\beta > 0$. The department fixed effects absorb the levels of contestation and land quality, so identification comes from the change in fertility within newly exposed departments relative to the change elsewhere.

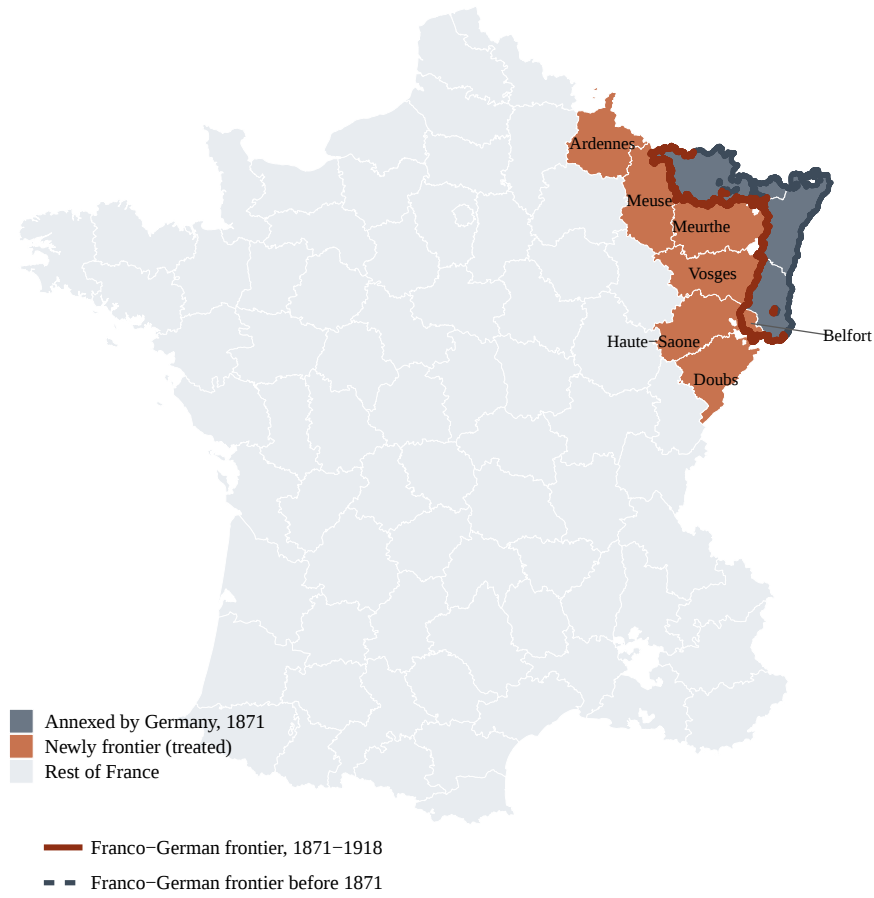


Figure 1: The frontier created in 1871. *Notes:* Department boundaries are those of 1790. The three departments annexed by Germany in 1871 — Bas-Rhin, Haut-Rhin and Moselle — are shown in grey and are excluded from all estimation. The seven departments shown in orange are those whose centroids lie within one hundred kilometres of the boundary of the annexed bloc, and constitute the treated group. The solid line marks the Franco-German frontier as it ran from 1871 to 1918; the dashed line marks the frontier before 1871, along the Rhine and the Prussian border. Territoire de Belfort is the arrondissement of Haut-Rhin that remained French and is shown separately.

One feature of the data requires an additional control. Fertility converges strongly over the nineteenth century: a regression of the 1806–1911 change in log fertility on its initial level yields a slope of -0.846 with an R^2 of 0.666 , so a department 10 percent higher in 1806 declined 8.5 percent further by 1911. Since the newly exposed departments were not at the mean of the initial distribution, a specification that ignores differential convergence risks attributing to the frontier a trajectory that initial conditions already imply. We therefore include period-specific slopes on 1806 log fertility, which allow the relationship between initial level and current fertility to differ freely across periods. This is more flexible than a linear convergence term and avoids a lagged dependent variable. We report estimates with and without it.

Inference deserves particular attention because treatment is assigned to only seven departments. We construct a placebo distribution by randomly reassigning the treatment indicator across departments one thousand times, holding the number of treated units fixed, and compare the estimate to that distribution. The placebo standard deviation is 0.039 against a clustered

standard error of 0.018, so the latter understates sampling variability by a factor of roughly two. On the permutation criterion the estimate in our preferred specification lies at the 91st percentile of the placebo distribution, corresponding to a two-sided p -value of 0.089. We report clustered standard errors in the tables for comparability and treat the permutation criterion as the basis for inference.

3.2.1 Baseline Difference-in-Differences Results

Table 1 reports the baseline difference-in-differences estimates of equation (10). All specifications include department and period fixed effects. We progressively augment the baseline specification by allowing for differential convergence, controlling for infant mortality, excluding the war year 1871, and excluding the Territoire de Belfort. Throughout, standard errors are clustered at the department level.

Table 1: Fertility and the Frontier Created in 1871

	(1)	(2)	(3)	(4)	(5)
<i>Panel A. All departments (74 controls)</i>					
Newly frontier $\times$ Post	0.094*** (0.020)	0.068*** (0.018)	0.037** (0.018)	0.080*** (0.018)	0.058*** (0.017)
Observations	1,780	1,780	1,588	1,699	1,759
Within R^2	0.014	0.012	0.105	0.016	0.007
<i>Panel B. Matched comparison group (23 controls)</i>					
Newly frontier $\times$ Post	0.085*** (0.013)	0.089*** (0.011)	0.070*** (0.012)	0.107*** (0.013)	0.087*** (0.013)
Observations	658	658	568	628	637
Within R^2	0.061	0.074	0.076	0.105	0.067
Department fixed effects	Yes	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	Yes	Yes	Yes
Convergence control		Yes	Yes	Yes	Yes
Infant mortality control			Yes		
Excludes 1871				Yes	
Excludes Belfort					Yes

Notes: The dependent variable is the log of the departmental fertility index. The reported coefficient is that on the interaction between an indicator for newly frontier departments and an indicator for periods from 1871 onward. The treated group comprises the 7 departments whose centroids lie within one hundred kilometers of the frontier created in 1871: Territoire de Belfort, Meurthe-et-Moselle, Meuse, Vosges, Haute-Saône, Doubs and Ardennes. The three annexed departments are excluded throughout. The convergence control consists of period-specific slopes on 1806 log fertility. Panel B retains, for each treated department, the 5 nearest comparison departments matched on 1806 and 1861 log fertility, the 1806–1866 fertility trend, and infant mortality. Standard errors clustered at the department level are reported in parentheses. Because treatment is assigned to only 7 departments, and because the comparison group in Panel B is selected using the characteristics of the treated departments, inference is based on permutation tests that reassign treatment across departments and, in Panel B, reconstruct the matched group on each draw. The permutation p -value for column 2 is 0.089 in Panel A and 0.019 in Panel B. Sequentially excluding each treated department from column 2 of Panel A yields coefficients between 0.058 and 0.078. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

The estimates consistently indicate that the creation of the new Franco-German frontier increased fertility in the newly exposed departments. In our preferred specification (column 2),

fertility increased by approximately 6.8 percent relative to the comparison departments following the 1871 annexation. The estimated effect remains economically large across all alternative specifications, ranging from 3.7 to 9.4 percent. It equals 9.4 percent in the specification with fixed effects only, 8.0 percent when the war year is excluded, 5.8 percent when the Territoire de Belfort is omitted, and 3.7 percent after controlling for infant mortality.

The estimated effect is also insensitive to the composition of the treated group. Re-estimating the preferred specification after sequentially excluding each of the seven treated departments yields coefficients between 5.8 and 7.8 percent, indicating that the results are not driven by any single department.

More broadly, the stability of the estimates across these alternative specifications is itself informative. The specifications differ in the controls they include, the sample composition, and the treatment period considered, yet the estimated effect remains consistently positive and of similar magnitude. This robustness suggests that the estimated increase in fertility is unlikely to reflect a particular specification choice or the influence of a single treated department.

3.2.2 Dynamic Effects

Fig. 2 examines the dynamics of the treatment effect by replacing the single post-treatment interaction in equation (10) with a full set of interactions between the treatment indicator and period dummies, normalizing the 1866 coefficient to zero. This event-study specification provides a direct assessment of the identifying assumption underlying the difference-in-differences design while illustrating the timing and persistence of the estimated effect.

The event-study estimates provide little evidence of differential pre-treatment trends. The twelve coefficients prior to 1871 fluctuate around zero with no systematic pattern. Ten are individually indistinguishable from zero, while the remaining two are negative rather than positive. Consistent with this visual evidence, a regression allowing for a linear differential trend over 1806–1866 yields a small and statistically insignificant coefficient of 0.0003 ($t = 0.04$). Together, these results support the parallel-trends assumption underlying the baseline difference-in-differences estimates.

The estimated treatment effect emerges immediately following the frontier reassignment and remains remarkably persistent. Fertility increases by 8.8 percent in 1876, the first observation after the annexation, remains close to ten percent throughout the following decades, and reaches its largest values around the turn of the century before declining to 9.2 percent in 1911. The absence of pre-treatment differences, together with the immediate and persistent post-treatment divergence, closely matches the timing implied by the historical shock and is consistent with the hypothesis that the increase in territorial contestability raised fertility in the newly exposed departments.

3.2.3 Alternative Comparison Groups

The baseline difference-in-differences estimates compare the newly exposed departments with the remainder of metropolitan France. While this comparison satisfies the identifying assumptions of the design, some regions provide more plausible counterfactuals than others. We therefore re-estimate the treatment effect using a matched comparison group constructed from departments with fertility trajectories most similar to those of the treated departments before 1871.

Specifically, for each treated department we retain the five nearest neighbors based on pre-treatment fertility levels (1806 and 1861), the pre-treatment fertility trend (1806–1866), and infant mortality, leaving a comparison group of 23 departments. We deliberately do not match

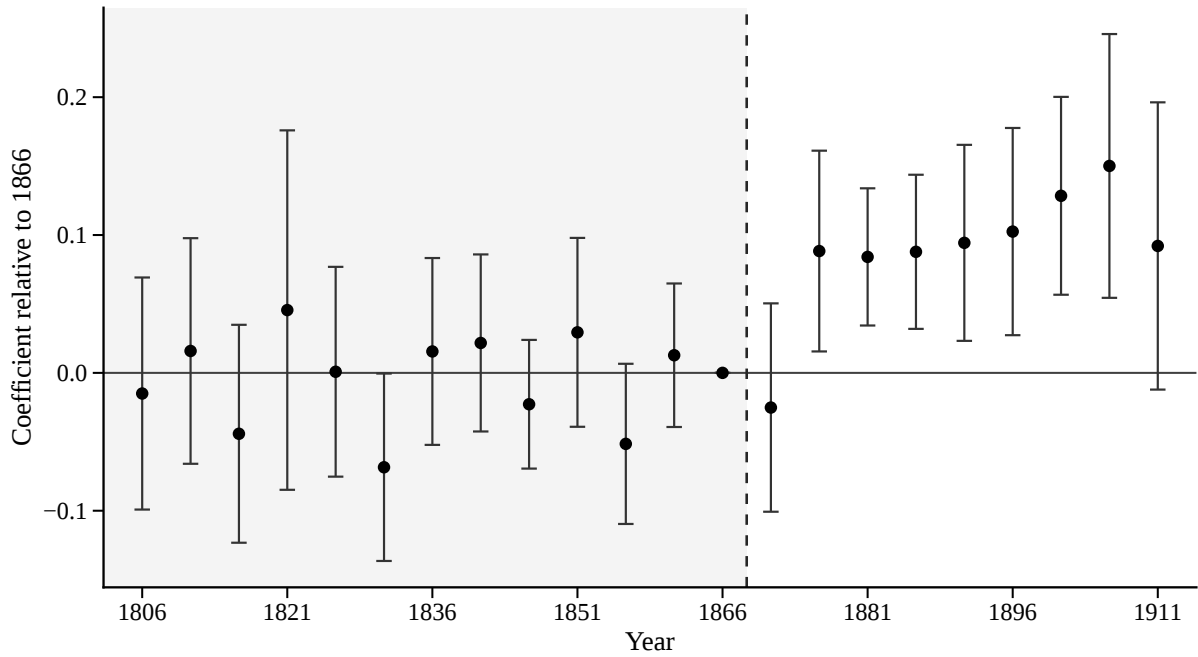


Figure 2: Dynamic effects of the 1871 frontier reassignment. *Notes:* The figure plots coefficients from a specification that replaces the single post-treatment interaction in equation (10) with a full set of interactions between the newly frontier indicator and period dummies, normalizing the 1866 coefficient to zero. The dependent variable is log fertility. The sample consists of 1,780 observations on 81 departments observed at quinquennial intervals between 1806 and 1911, excluding the annexed departments of Bas-Rhin, Haut-Rhin and Moselle. All estimates include department and period fixed effects. Solid circles denote point estimates and vertical bars denote 95 percent confidence intervals constructed from standard errors clustered at the department level. The shaded region marks the pre-treatment period and the vertical dashed line marks the 1871 annexation.

on land suitability or historical territorial contestability, since these variables provide the cross-sectional variation used later to evaluate the model's comparative statics. Likewise, we do not match on longitude because proximity to the new frontier is intrinsically related to treatment status and would mechanically restrict the comparison to neighboring departments that were themselves potentially affected by the geopolitical shock.

Restricting attention to more comparable control departments strengthens the estimated treatment effect. The preferred estimate increases from 6.8 percent in the full sample to 8.9 percent in the matched sample, and the estimated effect becomes more stable across alternative specifications. The difference is particularly evident in the specification controlling for infant mortality. Whereas this control reduces the estimated effect to 3.7 percent in the full sample, the corresponding estimate in the matched sample remains 7.0 percent. These results suggest that the baseline estimates are, if anything, attenuated by the inclusion of departments whose fertility dynamics differ systematically from those of the eastern frontier for reasons unrelated to the 1871 annexation.

The matched comparison also strengthens statistical inference. Because the estimated treatment effect increases while the dispersion of the permutation distribution changes little, the estimate lies further into the tail of the placebo distribution. The corresponding permutation p -value falls from 0.089 in the baseline specification to 0.019 in the matched sample.

The matched comparison group improves upon the baseline design by restricting attention to departments with similar pre-treatment characteristics. Given the long pre-treatment period

available in our data, we can go one step further and construct a comparison group that reproduces the treated departments' entire pre-1871 fertility trajectory. Fig. 3 implements this approach using the synthetic control method of Abadie et al. [2010], assigning non-negative weights across donor departments to minimize the discrepancy between the treated and synthetic groups prior to the 1871 annexation.

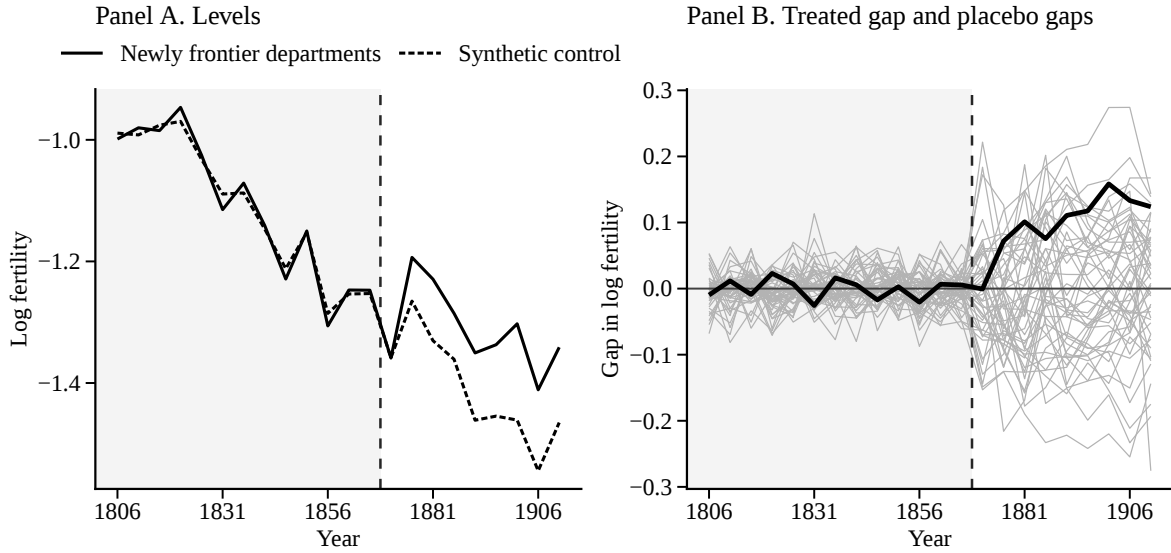


Figure 3: Synthetic control for the newly frontier departments. *Notes:* Panel (a) plots mean log fertility in the newly frontier departments against a synthetic control formed as a weighted average of the remaining departments, with weights chosen to minimize the squared difference in log fertility over 1806–1866 subject to being non-negative and summing to one. The root mean squared pre-treatment error is 0.014. Panel (b) plots the difference between the two series, together with the corresponding difference for each donor department when it is treated as if it had been exposed in 1871. Placebo donors whose own pre-treatment fit is more than three times worse than that of the treated unit are omitted, leaving 48 of 74. The shaded region marks the pre-treatment period and the vertical line marks the annexation.

The synthetic control closely reproduces the evolution of fertility in the treated departments before 1871. As shown in Panel (a), the two series are virtually indistinguishable throughout the pre-treatment period, with a root mean squared prediction error of only 0.014.² Following the annexation, however, the two series diverge immediately and remain separated for the rest of the sample. The average post-treatment gap is 9.9 percent, closely matching the estimate obtained from the matched difference-in-differences design and somewhat exceeding the baseline estimate.

Panel (b) evaluates the magnitude of this divergence relative to placebo estimates obtained by assigning treatment sequentially to each donor department. The treated gap remains essentially flat throughout the pre-treatment period and becomes one of the largest observed after 1871. The ratio of post- to pre-treatment fit equals 7.6, compared with a placebo median of 2.7, ranking seventh among the seventy-five units considered — the seventy-four donor departments and the treated group and corresponding to a permutation p -value of 0.093.

Taken together, the baseline difference-in-differences, matched comparison, and synthetic control analyses paint a remarkably consistent picture. Estimated treatment effects range from

²For comparison, fertility declines by roughly 0.35 log points over the sample period.

6.8 to 9.9 percent, with the two approaches that construct more comparable counterfactuals producing the larger estimates. Rather than weakening the evidence, increasingly demanding counterfactual comparisons strengthen it, suggesting that the baseline estimates are, if anything, conservative. Consistent with this interpretation, the synthetic control places most of its weight on departments such as Haute-Loire, Yonne, Haute-Garonne, Lot-et-Garonne, and Dordogne, none of which borders the treated region or was directly exposed to the geopolitical consequences of the 1871 annexation.

3.3 Heterogeneity Consistent with the Mechanism

The preceding analysis establishes that the creation of the new Franco-German frontier increased fertility in the newly exposed departments. The model, however, yields a sharper prediction than an average treatment effect. It predicts that the fertility response should be stronger where land remained economically important, since it is the strategic value of land that gives rise to the incentive to sustain larger populations. Conversely, where production had already shifted away from land-intensive activities, an increase in territorial contestability should have a smaller effect on fertility.

Table 2 evaluates these comparative statics by interacting the frontier shock with two predetermined department characteristics: agro-climatic land suitability and the industrial employment share in 1861, measured a decade before the annexation.

Table 2: Heterogeneity in the Fertility Response to Frontier Exposure

	(1)	(2)	(3)	(4)
Newly frontier $\times$ Post	0.065*** (0.020)	-0.054 (0.043)	0.070*** (0.016)	-0.053 (0.046)
$\times$ Land dependence		0.219*** (0.079)		0.218** (0.083)
$\times$ Industrialization, 1861			-0.007 (0.015)	-0.000 (0.006)
Observations	1,759	1,759	1,759	1,759
Within R^2	0.009	0.012	0.010	0.012
Department fixed effects	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	Yes	Yes
Convergence control	Yes	Yes	Yes	Yes

Notes: The dependent variable is the log of the departmental fertility index. Land dependence is standardized agro-climatic suitability and industrialization is the standardized share of the labor force in industrial occupations in 1861, both measured before the 1871 annexation. The model predicts a positive coefficient on the interaction with land dependence, since the incentive to sustain population is stronger where land contributes more to production, and a negative coefficient on the interaction with industrialization, since production that has already shifted away from land is less worth defending. The convergence control consists of period-specific slopes on 1806 log fertility. Standard errors clustered at the department level are reported in parentheses. The permutation p -value is 0.012 for the interaction with land dependence in column 2 and 0.885 for the interaction with industrialization in column 3. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

The evidence is broadly consistent with the mechanism. Column 2 shows that the interaction between frontier exposure and land suitability is positive, economically large, and statistically significant at the one-percent level. Once this interaction is introduced, the aver-

age treatment effect becomes small and statistically indistinguishable from zero, implying that the fertility response to the frontier reassignment was concentrated in departments where land remained relatively productive. This pattern is precisely what the model predicts: an increase in expropriation risk matters only where the asset being defended retains substantial economic value. Moreover, the result remains virtually unchanged after controlling simultaneously for pre-treatment industrialization in column 4.

The interaction with pre-1871 industrialization also carries the predicted negative sign in both specifications. Quantitatively, departments that had already industrialized appear to respond less to the increase in territorial contestability, consistent with the idea that industrialization reduced the strategic importance of land. The estimates, however, are small and imprecisely estimated, and we therefore view them as suggestive rather than conclusive evidence.³ Taken together, these results do not constitute a direct test of the full theoretical mechanism. Rather, they show that the heterogeneity of the causal effect induced by the 1871 frontier shock is broadly consistent with the comparative statics of the model.

3.4 Alternative Explanations

A natural concern is that the estimated increase in fertility reflects changes in the composition of the population rather than changes in fertility behavior. The annexation of Alsace-Lorraine displaced a substantial number of refugees, many of whom settled in departments adjacent to the new frontier. Because Alsace remained one of the highest-fertility regions in France during this period, the observed increase in fertility could simply reflect the arrival of high-fertility households rather than a behavioral response to the increase in territorial contestability.

This concern is *prima facie* plausible. The newly exposed departments received approximately 11.5 times more migrants from the annexed territories than the rest of France, and refugee intensity is strongly correlated with our treatment measure (correlation of 0.67). Table 3, however, provides little support for this interpretation.

Three findings are particularly informative. First, refugee intensity exhibits no independent relationship with fertility. When entered on its own in column 2, the estimated coefficient is close to zero and statistically insignificant. Second, controlling for refugee intensity strengthens rather than weakens the estimated frontier effect: the treatment coefficient increases from 0.058 to 0.066, while the refugee coefficient itself becomes negative. If refugee inflows were responsible for the increase in fertility, one would instead expect the estimated treatment effect to decline once migration is taken into account. Finally, the dynamic patterns of the two variables differ markedly. The estimated refugee effect peaks during the late 1880s before gradually disappearing, whereas the estimated frontier effect persists and even strengthens over time. These distinct dynamics are difficult to reconcile with a composition-based explanation.

One additional consideration deserves discussion. Between 1874 and 1880, France constructed the Séré de Rivières system of fortifications along the new frontier, bringing military investment, garrisons, and population movements to many of the same departments affected by the annexation. The construction of these fortifications is itself consistent with our interpretation: it reflects the increased strategic importance of the territory following the frontier reassignment and provides independent historical evidence that the annexation fundamentally altered the value of defending the eastern border. At the same time, the fortifications may

³The estimate is -0.007 with a clustered standard error of 0.015, and its permutation p -value is 0.885. The corresponding permutation p -value for the interaction with land suitability in column 2 is 0.012. With only seven treated departments there is little independent variation in either moderator, making it difficult to separate their roles in a sample of this size.

Table 3: Fertility, Frontier Exposure, and Refugee Settlement

	(1)	(2)	(3)
Newly frontier $\times$ Post	0.058*** (0.017)		0.066* (0.035)
Refugee share $\times$ Post		0.007 (0.008)	-0.003 (0.012)
Observations	1,759	1,759	1,759
Within R^2	0.007	0.002	0.008
Department fixed effects	Yes	Yes	Yes
Period fixed effects	Yes	Yes	Yes
Convergence control	Yes	Yes	Yes

Notes: The dependent variable is the log of the departmental fertility index. Refugee share is in-migration from the annexed territories as a share of departmental population in 1901, measured as the difference between total recorded in-migrants and in-migrants excluding Alsace-Lorraine, and is standardized. The newly exposed departments received approximately 11.5 times more migrants from the annexed territories than the remainder of France. The convergence control consists of period-specific slopes on 1806 log fertility. The sample is marginally smaller than in Table 1 because the migration series does not report the Territoire de Belfort separately. Standard errors clustered at the department level are reported in parentheses. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

have affected local demographic outcomes through channels beyond the behavioral response emphasized in our model, for example by increasing military employment or attracting migrants to frontier departments. With the available data, we cannot separately identify these complementary channels. Accordingly, our estimates should be interpreted as capturing the overall demographic consequences of the increase in the strategic importance of the frontier, rather than isolating the specific behavioral response to expropriation risk.

3.5 Cross-Sectional Evidence

The evidence thus far establishes that an exogenous increase in territorial contestability raises fertility and that this response is strongest where land remained economically important. The model, however, is ultimately a theory of the demographic transition. Its broader implication is that fertility should decline as industrialization erodes the strategic value of land. We therefore conclude the empirical analysis by asking whether long-run patterns of structural transformation across French departments are consistent with this prediction.

To do so, we estimate

$$\ln F_{rt} = \alpha_r + \delta_t + \beta_1 \text{Ind}_{rt} + \beta_2 (\text{Ind}_{rt} \times \text{Land}_r) + \beta_3 (\text{Ind}_{rt} \times \text{Border}_r) + \beta_4 (\text{Ind}_{rt} \times \text{Land}_r \times \text{Border}_r) + X'_{rt} \gamma + \varepsilon_{rt}, \quad (11)$$

where Ind_{rt} denotes the industrial employment share, Land_r is standardized agro-climatic suitability, and Border_r is standardized historical exposure to interstate conflict before 1700. The model predicts a negative coefficient on the triple interaction: as industrialization reduces the economic importance of land, fertility should decline more rapidly where land was both highly productive and historically more contestable. Because land suitability and historical contestability are time-invariant, they and their interaction are absorbed by the department fixed

effects. We therefore include the constituent two-way interactions throughout so that the triple interaction captures the additional heterogeneity implied by the model.

Table 4: Industrialization, Land Dependence, and Historical Contestation

	(1)	(2)	(3)	(4)
Industrialization	-0.377** (0.176)	-0.356** (0.142)	-0.331** (0.138)	-0.285** (0.127)
× Land dependence	0.039 (0.089)	0.049 (0.078)	-0.122 (0.139)	0.025 (0.068)
× Contestation	0.131* (0.078)	0.048 (0.079)	0.090 (0.120)	0.022 (0.072)
× Land dependence × Contestation	-0.069 (0.089)	-0.069 (0.080)	-0.046 (0.079)	-0.021 (0.069)
Observations	472	472	472	472
Within R^2	0.042	0.039	0.041	0.172
Department fixed effects	Yes	Yes	Yes	Yes
Period fixed effects	Yes	Yes	Yes	Yes
Convergence control		Yes	Yes	Yes
Land and contestation trends			Yes	
Infant mortality control				Yes

Notes: The dependent variable is the log of the departmental fertility index. Industrialization is the share of the labor force in industrial occupations, observed in six census years between 1851 and 1911. Land dependence and contestation are standardized agro-climatic suitability and standardized exposure to interstate conflict before 1700; their levels and their interaction are absorbed by the department fixed effects. The convergence control consists of period-specific slopes on 1806 log fertility, and the trend controls in column 3 add period-specific slopes on land dependence and contestation. Department and period fixed effects reduce the standard deviation of industrialization by roughly two-thirds, so that the minimum detectable triple interaction at conventional power is 0.224, or 63 percent of the estimated main effect. Standard errors clustered at the department level are reported in parentheses. *, **, and *** denote significance at the 10, 5, and 1 percent levels, respectively.

The results provide evidence broadly consistent with these predictions. Across all specifications, industrialization is associated with lower fertility, with estimated coefficients ranging from -0.285 to -0.377 and statistically significant at conventional levels. Using the preferred specification in column 2, a ten-percentage-point increase in the industrial employment share is associated with approximately a 3.6 percent decline in fertility.

The estimated triple interaction is negative in every specification, as the model predicts, although it is never estimated with sufficient precision to reject the null hypothesis of no effect. We interpret this lack of precision primarily as a consequence of limited identifying variation rather than evidence against the mechanism. Industrial employment is observed in only six census years, and after removing department and period fixed effects its standard deviation falls by roughly two-thirds, leaving relatively little within-department variation from which to identify heterogeneous responses.

Accordingly, we view this exercise as complementary to the evidence presented in Section 3.3. The 1871 frontier experiment provides credible causal evidence that an exogenous increase in territorial contestability raised fertility, particularly in land-dependent departments. The estimates reported here ask a different question: whether the broader process of industrialization exhibits the comparative-static patterns implied by the model. While the estimates are

necessarily imprecise, every interaction carries the predicted sign, and the overall pattern is consistent with the mechanism linking structural transformation, the declining strategic importance of land, and the demographic transition.

4 Conclusions

This paper develops a simple idea: when property rights are imperfectly enforced, population serves not only productive purposes but also strategic ones. Societies have an incentive to sustain larger populations because population increases their ability to defend valuable land from external appropriation. As industrialization reduces the economic importance of land, this strategic return to population declines, weakening one incentive for maintaining high fertility. The mechanism therefore suggests a novel channel through which structural transformation may have contributed to the demographic transition.

We evaluate this hypothesis using a panel of French departments spanning 1806–1911. Our central empirical strategy exploits the 1871 annexation of Alsace-Lorraine, which unexpectedly shifted the Franco-German frontier westward and increased the strategic importance of several previously interior departments. We find that fertility increased by approximately seven percent in these newly exposed departments relative to the rest of France. This result is robust across a variety of specifications, exhibits no evidence of differential pre-trends, survives alternative constructions of the comparison group, and cannot be explained by the settlement of refugees from the annexed territories.

The historical experiment also provides evidence on the mechanism itself. The fertility response is concentrated in departments where land remained relatively productive, precisely where the model predicts that an increase in territorial contestability should matter most. We also document broader cross-sectional patterns consistent with the model’s comparative statics: industrialization is associated with lower fertility throughout the sample, and the estimated interactions between industrialization, land dependence, and historical contestability uniformly carry the signs implied by the theory, although they are estimated imprecisely. We view these latter results as suggestive rather than conclusive evidence that the strategic fertility channel documented by the natural experiment may also have contributed to the long-run demographic transition.

Our findings contribute to the literature in two ways. First, they provide causal evidence that exogenous changes in the strategic value of land can influence fertility behavior. Second, they suggest that this strategic motive weakened as industrialization reduced both the productive and strategic importance of land. While the evidence does not imply that this mechanism alone explains the demographic transition, it identifies a channel that has received little attention in the existing literature and that appears capable of accounting for an important part of the historical decline in fertility.

Several limitations remain. The industrialization analysis relies on relatively limited within-department variation and therefore cannot provide evidence as sharp as the 1871 natural experiment. In addition, the military fortifications constructed along the new frontier after 1874 likely formed part of the broader response to the increased strategic importance of the eastern border. Although this historical response is itself consistent with the mechanism we propose, the available data do not allow us to distinguish the direct behavioral response to increased territorial contestability from other demographic consequences associated with frontier militarization. These limitations notwithstanding, the evidence consistently points toward the existence of a strategic component of fertility decisions whose importance declined as Europe underwent industrialization and structural transformation.

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